Linear algebra reviews exercises

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Problem 1:

Do the three planes $2x_1 + 4x_2 + 4_3 = 4$, $x_2 - 2x_3 = -2$ and $2x_1 + 3x_2 = 0$ have at least one common point of intersection? Explain.

Problem 2:

Find a general solution in vector parametric form and give a geometric description to the system whose augmented matrices are:

1. $\begin{pmatrix} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{pmatrix}$,
2. $\begin{pmatrix} 3 & -2 & 4 & 0 \\ 9 & -6 & 12 & 0 \\ 6 & -4 & 8 & 0 \end{pmatrix}$,
3. $\begin{pmatrix} 1 & 0 & -9 & 0 & 4 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 0 & -7 \end{pmatrix}$.

Problem 3:

Let $A = \begin{pmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ -7 \\ -3 \end{pmatrix}$.

1. Is b in the span of the columns of A?
2. Does the columns of A span $\mathbb{R}^n$?
3. Is the matrix transformation $x \mapsto Ax$ onto $\mathbb{R}^3$?

Problem 4:

Balance the chemical equation using the vector equation approach. Boron sulfide reacts violently with water to form boric acid and hydrogen sulfide gas (the smell of rotten eggs). The unbalanced equation is:

$$B_2S_3 + H_2O \leftarrow H_3BO_3 + H_2S$$

Problem 5:

Determine if the columns of the matrix form a linearly independent set.

$$A = \begin{pmatrix} -4 & -3 & 0 \\ 0 & -1 & 5 \\ 1 & 1 & -5 \\ 2 & 1 & -10 \end{pmatrix}$$

Is the matrix transformation $x \mapsto Ax$ one-to-one?

Problem 6:
$T : \mathbb{R}^2 \to \mathbb{R}^2$ first reflects points through the horizontal x_1-axis and then reflect points through the line $x_2 = -x_1$.

Problem 7: Compute the inverse using the algorithm seen in class.

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Problem 8:
Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. Show that T is invertible and find a formula for T^{-1}.

$$T(x_1, x_2) = (-5x_1 + 9x_2, 4x_1 - 7x_2).$$

Problem 9:
Solve the equation $Ax = b$, by using the LU factorization given for A.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{pmatrix} \begin{pmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{pmatrix}$$

and

$$b = \begin{pmatrix} -7 \\ 5 \\ 2 \end{pmatrix}$$

Problem 9:
Use the cofactor expansion to compute

$$\begin{vmatrix} 5 & -2 & 4 \\ 0 & 3 & -5 \\ 2 & -4 & 7 \end{vmatrix}$$

Problem 10:

Use row reduction to echelon form to compute the following determinant

$$\begin{vmatrix} 1 & a & a_2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Problem 11:

Using the Cramer's rule, determine the values of the parameter for which the system has a unique solution, and describe the solution.

$$\begin{cases} sx_1 - 2sx_2 & = & 1 \\ 3x_1 + 6sx_2 & = & 4 \end{cases}$$

Problem 12:

Let R be the triangle with vertices at $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3). Show that

$$\{area\ of\ triangle\} = 1/2 det \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix}$$